Lecture 06
12.5: Distance with lines and planes

Jeremiah Southwick

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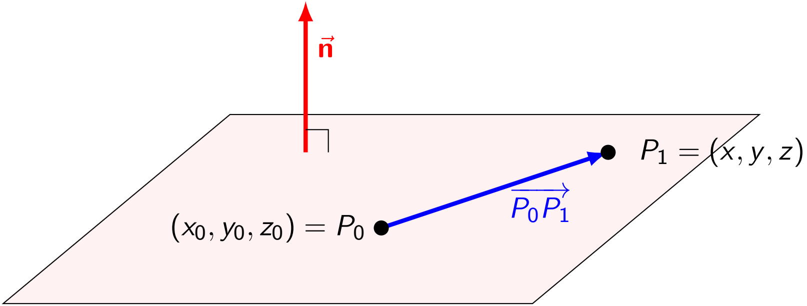
Things to note

Office hours today: 12-2

Quiz average: 8.33 (without 0's) Quiz average: 6.76 (with 0's)

Collect HW2.

Last Class



Definition

Let $\vec{\mathbf{n}} = \langle A, B, C \rangle$ *be a normal vector to a plane containing the point* $P_0 = (x_0, y_0, z_0)$*. Then the equation of the plane (where* $P_1 = (x, y, z)$*) is*

$$\vec{\mathbf{n}} \cdot \overrightarrow{P_0P_1} = 0$$

or

$$Ax + By + Cz = Ax_0 + By_0 + Cz_0 \text{ the coordinate form simplified.}$$

Plane example

Example

Find the equation of the plane through $R = (0, 0, 1)$, $S = (2, 0, 0)$, and $T = (0, 3, 0)$.

Two vectors in the plane are $\overrightarrow{RS} = \langle 2, 0, -1 \rangle$ and $\overrightarrow{RT} = \langle 0, 3, -1 \rangle$.

$$\overrightarrow{RS} \times \overrightarrow{RT} = \begin{vmatrix} \vec{\mathbf{i}} & \vec{\mathbf{j}} & \vec{\mathbf{k}} \\ 2 & 0 & -1 \\ 0 & 3 & -1 \end{vmatrix} = \begin{vmatrix} 0 & -1 \\ 3 & -1 \end{vmatrix} \vec{\mathbf{i}} - \begin{vmatrix} 2 & -1 \\ 0 & -1 \end{vmatrix} \vec{\mathbf{j}} + \begin{vmatrix} 2 & 0 \\ 0 & 3 \end{vmatrix} \vec{\mathbf{k}} = \langle 3, 2, 6 \rangle.$$

Plane example

Example

Find the equation of the plane through $R = (0, 0, 1)$*,* $S = (2, 0, 0)$*, and* $T = (0, 3, 0)$*.*

Two vectors in the plane are $\overrightarrow{RS} = \langle 2, 0, -1\rangle$ and $\overrightarrow{RT} = \langle 0, 3, -1\rangle$.

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Thus the equation of the plane is

$$\langle 3, 2, 6\rangle \cdot \langle x - 0, y - 0, z - 1\rangle = 0, \text{ or } 3x + 2y + 6z = 6.$$

Notice you could use any of the given points.

Combining lines and planes

Example

Find the line of intersection of the planes $3x - 6y - 2z = 15$ *and* $2x + y - 2z = 5$.

Direction:

Combining lines and planes

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$$\overrightarrow{n_1} \times \overrightarrow{n_2} = \begin{vmatrix} \vec{\mathbf{i}} & \vec{\mathbf{j}} & \vec{\mathbf{k}} \\ 3 & -6 & -2 \\ 2 & 1 & -2 \end{vmatrix} = \begin{vmatrix} -6 & -2 \\ 1 & -2 \end{vmatrix} \vec{\mathbf{i}} - \begin{vmatrix} 3 & -2 \\ 2 & -2 \end{vmatrix} \vec{\mathbf{j}} + \begin{vmatrix} 3 & -6 \\ 2 & 1 \end{vmatrix} \vec{\mathbf{k}} = \langle 14, 2, 15 \rangle.$$

Point:

Combining lines and planes

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Point: When $z = 0$,

$$3x - 6y = 15 \text{ and } 2x + y = 5 \Rightarrow 15x + 0y = 45 \Rightarrow x = 3, y = -1$$

Line:

Combining lines and planes

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Point: When $z = 0$,

$$3x - 6y = 15 \text{ and } 2x + y = 5 \Rightarrow 15x + 0y = 45 \Rightarrow x = 3, y = -1$$

Line: $\vec{\mathbf{r}}(t) = \langle 3, -1, 0 \rangle + t \langle 14, 2, 15 \rangle$.

Combining lines and planes, cont.

Example

Find the point of intersection between the line $\vec{\mathbf{r}}(t) = \langle \frac{8}{3} + 2t, -2t, 1 + t \rangle$ *and the plane* $3x + 2y + 6z = 6$.

Combining lines and planes, cont.

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Find the point of intersection between the line $\vec{\mathbf{r}}(t) = \langle \frac{8}{3} + 2t, -2t, 1 + t \rangle$ *and the plane* $3x + 2y + 6z = 6$.

$$3(\frac{8}{3} + 2t) + 2(-2t) + 6(1 + t) = 6$$

$$\Rightarrow 8 + 6t - 4t + 6 + 6t = 6 \Rightarrow 8t = -8 \Rightarrow t = -1.$$

Combining lines and planes, cont.

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Find the point of intersection between the line $\vec{\mathbf{r}}(t) = \langle \frac{8}{3} + 2t, -2t, 1 + t \rangle$ *and the plane* $3x + 2y + 6z = 6$*.*

$$3(\frac{8}{3} + 2t) + 2(-2t) + 6(1 + t) = 6$$

$$\Rightarrow 8 + 6t - 4t + 6 + 6t = 6 \Rightarrow 8t = -8 \Rightarrow t = -1.$$

So the point is $(\frac{8}{3} + 2(-1), -2(-1), 1 - 1) = (\frac{2}{3}, 2, 0)$.

Distance Activity

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3b. "Does that make sense?"

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3a. "How did you figure that out?"

3b. "Does that make sense?"

3c. "What made you think to do that?"

4. Activity will **not** be collected
5. Raise hand to get Jeremiah's attention if you need it